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important facts in astronomy, and the laws of light and vision, branches wonderfully calculated to arrest and occupy the attention, in certain stages of mental development, especially when faith in the universality of God's providence goes with them.

The respect and companionship, in a degree I never could have anticipated, of those who dwell by the fountains of knowledge, have cheered me; men whose virtues and accomplishments I can never think to emulate, and whose envy, if they had any, could not descend upon me.

Mr. President, I have said nothing of methods. When first questioned by the Rumford Committee as to what was original in my mode of working, I proposed that they should visit my shop, where I could show and explain to them the course pursued, and they might judge for themselves of the value or originality of any part of it. They have done so; and I accept their decision, trusting the egotism displayed in this little history of achievements may be excused.

You will not fail to perceive, that, after so many years of hopeful, cheerful, and patient toil, mingled with no ordinary share of painstaking, a high appreciation on the part of the recipient must follow this award, as naturally as light comes with the rising of the sun.

Five hundred and eightieth Meeting.

March 12, 1867. — ADJOURNED STATUTE MEETING.

The PRESIDENT in the chair.

The President called the attention of the Academy to the necessity of some action for the accommodation of the Academy, and proposed that a committee be appointed to consider this subject.

On the motion of Professor Rogers, it was voted, that a committee of three be appointed, and on the nomination of the President, Colonel Lyman, Mr. C. M. Warren, and Professor C. W. Eliot were chosen.

On the motion of Dr. Clark, this committee were instructed to inquire concerning the co-operation of other societies in providing a building for their common accommodation.

The following paper was presented.

On an Improvement in Boole's Calculus of Logic. By C. S. PEIRCE.

THE principal use of Boole's Calculus of Logic lies in its application to problems concerning probability. It consists, essentially, in a system of signs to denote the logical relations of classes. The data of any problem may be expressed by means of these signs, if the letters of the alphabet are allowed to stand for the classes themselves. From such expressions, by means of certain rules for transformation, expressions can be obtained for the classes (of events or things) whose frequency is sought in terms of those whose frequency is known. Lastly, if certain relations are known between the logical relations and arithmetical operations, these expressions for events can be converted into expressions for their probability.

It is proposed, first, to exhibit Boole's system in a modified form, and second, to examine the difference between this form and that given by Boole himself.

Let the letters of the alphabet denote classes whether of things or of occurrences. It is obvious that an event may either be singular, as "this sunrise," or general, as "all sunrises." Let the sign of equality with a comma beneath it express numerical identity. Thus $a \underset{,}{=} b$ is to mean that a and b denote the same class,—the same collection of individuals.

Let $a +_{,} b$ denote all the individuals contained under a and b together. The operation here performed will differ from arithmetical addition in two respects: 1st, that it has reference to identity, not to equality; and 2d, that what is common to a and b is not taken into account twice over, as it would be in arithmetic. The first of these differences, however, amounts to nothing, inasmuch as the sign of identity would indicate the distinction in which it is founded; and therefore we may say that

(1.) If No a is b $\quad a +_{,} b \underset{,}{=} a + b$

It is plain that

(2.) $\quad a +_{,} a \underset{,}{=} a$

and also, that the process denoted by $+_{,}$, and which I shall call the process of *logical addition,* is both commutative and associative. That is to say

(3.) $\quad a +_{,} b \underset{,}{=} b +_{,} a$

and

(4.) $$(a +_{,} b) +_{,} c =_{,} a +_{,} (b +_{,} c).$$

Let $a\,,b$ denote the individuals contained at once under the classes a and b; those of which a and b are the common species. If a and b were independent events, $a\,,b$ would denote the event whose probability is the product of the probabilities of each. On the strength of this analogy, (to speak of no other,) the operation indicated by the comma may be called logical multiplication. It is plain that

(5.) $$a\,,a =_{,} a.$$

Logical multiplication is evidently a commutative and associative process. That is,

(6.) $$a\,,b =_{,} b\,,a$$

(7.) $$(a\,,b)\,,c = a\,,(b\,,c).$$

Logical addition and logical multiplication are doubly distributive, so that

(8.) $$(a +_{,} b)\,,c =_{,} a\,,c +_{,} b\,,c$$

and

(9.) $$a\,,b +_{,} c =_{,} (a +_{,} c)\,,(b +_{,} c).$$

Proof. Let
$$a =_{,} a' + x + y + o$$
$$b =_{,} b' + x + z + o$$
$$c =_{,} c' + y + z + o$$

where any of these letters may vanish. These formulæ comprehend every possible relation of $a\,,b$ and c; and it follows from them, that

$$a +_{,} b =_{,} a' + b' + x + y + z + o \qquad (a +_{,} b)\,,c =_{,} y + z + o.$$

But

$$a\,,c =_{,} y + o \qquad b\,,c =_{,} z + o \qquad a\,,c +_{,} b\,,c =_{,} y + z + o \quad \therefore (8).$$

So

$$a\,,b =_{,} x + o \qquad a\,,b +_{,} c =_{,} c' + x + y + z + o.$$

But

$$(a +_{,} c) = a' + c' + x + y + z + o \qquad (b +_{,} c) =_{,} b' + c' + x + y + z + o$$
$$(a +_{,} c)\,,(b +_{,} c) =_{,} c' + x + y + z + o \quad \therefore (9).$$

Let $-_{,}$ be the sign of logical subtraction; so defined that

(10.) If $b +_{,} x =_{,} a \qquad x =_{,} a -_{,} b.$

Here it will be observed that x is not completely determinate. It may vary from a to a with b taken away. This minimum may be denoted by $a - b$. It is also to be observed that if the sphere of b reaches at all beyond a, the expression $a -_{,} b$ is uninterpretable. If then we denote the contradictory negative of a class by the letter which denotes the class itself, with a line above it,* if we denote by v a wholly indeterminate class, and if we allow $[0 -_{,} 1]$ to be a wholly uninterpretable symbol, we have

(11.) $a -_{,} b =_{,} v , a , b + a , \bar{b} + [0 -_{,} 1] , \bar{a} , b$

which is uninterpretable unless

$$\bar{a} , b =_{,} 0.$$

If we define zero by the following identities, in which x may be any class whatever,

(12.) $0 =_{,} x -_{,} x =_{,} x - x$

then, *zero* denotes the class which does not go beyond any class, that is *nothing* or nonentity.

Let $a ; b$ be read a logically divided by b, and be defined by the condition that

(13.) If $b , x =_{,} a \qquad x =_{,} a ; b$

x is not fully determined by this condition. It will vary from a to $a + \bar{b}$ and will be uninterpretable if a is not wholly contained under b. Hence, allowing $[1 ; 0]$ to be some uninterpretable symbol,

(14.) $a ; b =_{,} a , b + v , \bar{a} , \bar{b} + [1 ; 0] \; a , \bar{b}$

which is uninterpretable unless

$$a , \bar{b} =_{,} 0.$$

Unity may be defined by the following identities in which x may be any class whatever.

(15.) $1 =_{,} x ; x =_{,} x : x.$

Then *unity* denotes the class of which any class is a part; that is, *what is* or *ens*.

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* So that, for example, $\bar{a}$ denotes not-a.

It is plain that if for the moment we allow $a:b$ to denote the maximum value of $a ; b$, then

(16.) $$\bar{x} \underset{,}{=} 1 - x \underset{,}{=} 0:x.$$

So that

(17.) $$x , (1 - x) \underset{,}{=} 0 \qquad x +_{,} 0:x \underset{,}{=} 1.$$

The rules for the transformation of expressions involving logical subtraction and division would be very complicated. The following method is, therefore, resorted to.

It is plain that any operations consisting solely of logical addition and multiplication, being performed upon interpretable symbols, can result in nothing uninterpretable. Hence, if $\varphi + \times\, x$ signifies such an operation performed upon symbols of which x is one, we have

$$\varphi + \times\, x \underset{,}{=} a , x + b , (1 - x)$$

where a and b are interpretable.

It is plain, also, that all four operations being performed in any way upon any symbols, will, in general, give a result of which one term is interpretable and another not; although either of these terms may disappear. We have then

$$\varphi x \underset{,}{=} i , x + j , (1 - x).$$

We have seen that if either of these coefficients i and j is uninterpretable, the other factor of the same term is equal to nothing, or else the whole expression is uninterpretable. But

$$\varphi\,(1) \underset{,}{=} i \text{ and } \varphi\,(0) \underset{,}{=} j.$$

Hence

(18.) $$\varphi x \underset{,}{=} \varphi\,(1) , x + \varphi\,(0) , (1 - x)$$

$$\varphi\,(x \text{ and } y) \underset{,}{=} \varphi\,(1 \text{ and } 1) , x , y + \varphi\,(1 \text{ and } 0) , x , \bar{y} + \varphi\,(0 \text{ and } 1) , \bar{x} , y + \varphi\,(0 \text{ and } 0) , \bar{x} , \bar{y}.$$

(18'.) $$\varphi x \underset{,}{=} (\varphi\,(1) +_{,} \bar{x}) , (\varphi\,(0) +_{,} x)$$

$$\varphi\,(x \text{ and } y) \underset{,}{=} (\varphi\,(1 \text{ and } 1) +_{,} \bar{x} +_{,} \bar{y}) , (\varphi\,(1 \text{ and } 0) +_{,} \bar{x} +_{,} y) , (\varphi\,(0 \text{ and } 1) +_{,} x +_{,} \bar{y}) , (\varphi\,(0 \text{ and } 0) +_{,} x +_{,} y).$$

Developing by (18) $x \underset{,}{-} y$, we have,

$$x \underset{,}{-} y \underset{,}{=} (1 \underset{,}{-} 1) , x , y + (1 \underset{,}{-} 0) , x , \bar{y} + (0 \underset{,}{-} 1) , \bar{x} , y + (0 \underset{,}{-} 0) , \bar{x} , \bar{y}.$$

So that, by (11),

(19.) $$(1 \underset{,}{-} 1) \underset{,}{=} v \quad 1 \underset{,}{-} 0 \underset{,}{=} 1 \quad 0 \underset{,}{-} 1 \underset{,}{=} [0 \underset{,}{-} 1] \quad 0 \underset{,}{-} 0 \underset{,}{=} 0.$$

Developing $x;y$ in the same way, we have*

$$x;y =_{,} 1;1,x,y + 1;0,x,\bar{y} + 0;1,\bar{x},y + 0;0,\bar{x},\bar{y}.$$

So that, by (14),

(20.) $1;1 =_{,} 1 \qquad 1;0 =_{,} [1;0] \qquad 0;1 =_{,} 0 \qquad 0;0 =_{,} v.$

Boole gives (20), but not (19).

In solving identities we must remember that

(21.) $$(a +_{,} b) - b =_{,} a$$

(22.) $$(a -_{,} b) +_{,} b =_{,} a.$$

From $a -_{,} b$ the value of b cannot be obtained.

(23.) $$(a,b) \div b =_{,} a$$

(24.) $$a;b,b =_{,} a.$$

From $a;b$ the value of b cannot be determined.

Given the identity $\qquad \varphi\, x =_{,} 0.$

Required to eliminate x.

$$\varphi\,(1) =_{,} x,\varphi\,(1) + (1 - x),\varphi\,(1)$$

$$\varphi\,(0) =_{,} x,\varphi\,(0) + (1 - x),\varphi\,(0).$$

Logically multiplying these identities, we get

$$\varphi\,(1),\varphi\,(0) =_{,} x,\varphi\,(1),\varphi\,(0) + (1 - x),\varphi\,(1),\varphi\,(0).$$

For two terms disappear because of (17).

But we have, by (18),

$$\varphi\,(1),x + \varphi\,(0),(1 - x) =_{,} \varphi\, x =_{,} 0.$$

Multiplying logically by x we get

$$\varphi\,(1),x =_{,} 0$$

and by $(1 - x)$ we get

$$\varphi\,(0),(1 - x) =_{,} 0.$$

Substituting these values above, we have

(25.) $$\varphi\,(1),\varphi\,(0) =_{,} 0 \text{ when } \varphi\, x =_{,} 0.$$

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* $a;b,c$ must always be taken as $(a;b),c$, not as $a;(b,c)$.

Given $\varphi\, x =_{,} 1.$

Required to eliminate x.

Let $\varphi'\, x =_{,} 1 - \varphi\, x =_{,} 0$

$$\varphi'\,(1)\,,\varphi'\,(0) =_{,} (1-\varphi\,(1))\,,(1-\varphi\,(0)) =_{,} 0$$

$$1-(1-\varphi\,(1))\,,(1-\varphi\,(0)) =_{,} 1.$$

Now, developing as in (18), only in reference to $\varphi\,(1)$ and $\varphi\,(0)$ instead of to x and y,

$$1-(1-\varphi\,(1))\,,(1-\varphi\,(0)) =_{,} \varphi\,(1)\,,\varphi\,(0) + \varphi\,(1)\,,(1-\varphi\,(0)) + \varphi\,(0)\,,(1-\varphi\,(1)).$$

But by (18) we have also,

$$\varphi\,(1) +_{,} \varphi\,(0) =_{,} \varphi\,(1)\,,\varphi\,(0) + \varphi\,(1)\,,(1-\varphi\,(0)) + \varphi\,(0)\,,(1-\varphi\,(1)).$$

So that

(26.) $\varphi\,(1) +_{,} \varphi\,(0) =_{,} 1$ when $\varphi\, x =_{,} 1.$

Boole gives (25), but not (26).

We pass now from the consideration of *identities* to that of *equations.*

Let every expression for a class have a second meaning, which is its meaning in an equation. Namely, let it denote the proportion of individuals of that class to be found among all the individuals examined in the long run.

Then we have

(27.) If $a =_{,} b \qquad a = b$

(28.) $a + b = (a +_{,} b) + (a\,,b).$

Let b_a denote the frequency of b's among the a's. Then considered as a class, if a and b are events b_a denotes the fact that if a happens b happens.

(29.) $a\,b_a = a\,,b.$

It will be convenient to set down some obvious and fundamental properties of the function b_a.

(30.) $a\,b_a = b\,a_b$

(31.) $\varphi\,(b_a \text{ and } c_a) = (\varphi\,(b \text{ and } c))_a$

(32.) $(1-b)_a = 1 - b_a$

(33.) $$b_{\mathrm{a}} = \frac{b}{a} + b_{(1-\mathrm{a})}\left(1 - \frac{1}{a}\right)$$

(34.) $$a_{\mathrm{b}} = 1 - \frac{1-a}{b}\, b_{(1-\mathrm{a})}$$

(35.) $$(\varphi a)_{\mathrm{a}} = (\varphi\,(1))_{\mathrm{a}}.$$

The application of the system to probabilities may best be exhibited in a few simple examples, some of which I shall select from Boole's work, in order that the solutions here given may be compared with his.

Example 1. Given the proportion of days upon which it hails, and the proportion of days upon which it thunders. Required the proportion of days upon which it does both.

Let $1 \underset{,}{=}$ days,
$p \underset{,}{=}$ days when it hails,
$q \underset{,}{=}$ days when it thunders,
$r \underset{,}{=}$ days when it hails and thunders.

$$p\,, q \underset{,}{=} r$$

Then by (29), $$r \underset{,}{=} p\,, q = p\, q_{\mathrm{p}} = q\, p_{\mathrm{q}}.$$

Answer. The required proportion is an unknown fraction of the least of the two proportions given.

By p might have been denoted the probability of the major, and by q that of the minor premise of a hypothetical syllogism of the following form: —

If a noise is heard, an explosion always takes place;
If a match is applied to a barrel of gunpowder, a noise is heard;
∴ *If a match is applied to a barrel of gunpowder, an explosion always takes place.*

In this case, the value given for r would have represented the probability of the conclusion. Now Boole (p. 284) solves this problem by his unmodified method, and obtains the following answer: —

$$r = p\,q + a\,(1 - q)$$

where a is an arbitrary constant. Here, if $q = 1$ and $p = 0$, $r = 0$. That is, his answer implies that if the major premise be false and the minor be true, the conclusion must be false. That this is not really so is shown by the above example. Boole (p. 286) is forced to the conclusion that "propositions which, when true, are equivalent, are not necessarily equivalent when regarded only as probable." This is absurd, because probability belongs to the events denoted, and not to forms of expression. The probability of an event is not altered by translation from one language to another.

Boole, in fact, puts the problem into equations wrongly (an error which it is the chief purpose of a calculus of logic to prevent), and proceeds as if the problem were as follows: —

It being known what would be the probability of Y, if X were to happen, and what would be the probability of Z, if Y were to happen; what would be the probability of Z, if X were to happen?

But even this problem has been wrongly solved by him. For, according to his solution, where

$$p = Y_{\mathrm{X}} \qquad q = Z_{\mathrm{Y}} \qquad r = Z_{\mathrm{X}},$$

r must be at least as large as the product of p and q. But if X be the event that a certain man is a negro, Y the event that he is born in Massachusetts, and Z the event that he is a white man, then neither p nor q is *zero*, and yet r vanishes.

This problem may be rightly solved as follows: —

$$\text{Let } p' \underset{,}{=} Y_{\mathrm{p}} \underset{,}{=} X, Y$$

$$q' \underset{,}{=} Z_{\mathrm{q}} \underset{,}{=} X, Z$$

$$r' \underset{,}{=} Z_{\mathrm{r}} \underset{,}{=} X, Z.$$

$$\text{Then, } r' \underset{,}{=} p', q'; p' \underset{,}{=} p', q'; q'.$$

Developing these expressions by (18) we have

$$r' \underset{,}{=} p', q' + r'_{\mathrm{p}', \bar{\mathrm{q}}'} (p', \bar{q}') + r'_{\bar{\mathrm{p}}', \bar{\mathrm{q}}'} (\bar{p}', \bar{q}')$$

$$\underset{,}{=} p', q' + r'_{\bar{\mathrm{p}}', \mathrm{q}'} (\bar{p}', q') + r'_{\bar{\mathrm{p}}', \bar{\mathrm{q}}'} (p', q').$$

The comparison of these two identities shows that

$$r' \underset{,}{=} p', q' + r'_{\bar{\mathrm{p}}', \bar{\mathrm{q}}'} (\bar{p}', \bar{q}').$$

$$\text{Let } V \underset{,}{=} r'_{\bar{\mathrm{p}}', \bar{\mathrm{q}}'} \underset{,}{=} \frac{x, \bar{y}, z}{\bar{x}, y, \bar{z} + \bar{y}}$$

Now $$p', q' \underset{,}{=} p' - p', \bar{q}' \underset{,}{=} q' - q', \bar{p}'$$

$$\bar{p}', \bar{q}' \underset{,}{=} \bar{q}' - p', \bar{q}' \underset{,}{=} \bar{p}' - q', \bar{p}'$$

And $$p', \bar{q}' \underset{,}{=} p' - p'_{q'}\, q' \underset{,}{=} \bar{q}' - \bar{q}'_{\bar{p}'}\, \bar{p}'$$

$$\bar{p}', q' \underset{,}{=} q' - q'_{p'}\, p' \underset{,}{=} \bar{p}' - \bar{p}'_{\bar{q}'}\, \bar{q}'$$

Then let

$$A \underset{,}{=} p'_{q'} \underset{,}{=} \frac{x, y, z}{y, z}$$

$$B \underset{,}{=} \bar{q}'_{\bar{p}'} \underset{,}{=} \frac{\bar{x}, y, \bar{z} + \bar{x}, \bar{y}, z + x, \bar{y}, z + \bar{x}, \bar{y}, \bar{z}}{1 - x, y}$$

$$C \underset{,}{=} \bar{p}'_{\bar{q}'} \underset{,}{=} \frac{\bar{x}, y, \bar{z} + \bar{x}, \bar{y}, z + x, \bar{y}, z + \bar{x}, \bar{y}, \bar{z}}{1 - y, z}$$

$$D \underset{,}{=} q'_{p'} \underset{,}{=} \frac{x, y, z}{x, y}$$

And we have

$$r = \frac{Y}{Z}p + V\left(\frac{1}{Z} - q\right) - (1 + V)\left(\frac{Y}{Z}p - Aq\right)$$

$$= \frac{Y}{Z}p + V\left(\frac{1}{Z} - q\right) - (1 + V)\left(\frac{1}{Z} - q - B\left(\frac{1 - Yp}{Z}\right)\right)$$

$$= q + V\left(\frac{1 - Yp}{Z}\right) - (1 + V)\left(\frac{1 - Yp}{Z} - C\left(\frac{1}{Z} - q\right)\right)$$

$$= q + V\left(\frac{1 - Yp}{Z}\right) - (1 + V)\left(q - D\frac{Y}{Z}p\right)$$

Ex. 2. (See Boole, p. 276.) Given r and q; to find p.

$p \underset{,}{=} r ; q \underset{,}{=} r + v, (1 - q)$ because p is interpretable.

Ans. The required proportion lies somewhere between the proportion of days upon which it both hails and thunders, and that added to one minus the proportion of days when it thunders.

Ex. 3. (See Boole, p. 279.) Given, out of the number of questions put to two witnesses, and answered by *yes* or *no*, the proportion that each answers truly, and the proportion of those their answers to which disagree. Required, out of those wherein they agree, the proportion they answer truly and the proportion they answer falsely.

Let $1 \underset{,}{=}$ the questions put to both witnesses,

$p \underset{,}{=}$ those which the first answers truly,

$q \underset{,}{=}$ those which the second answers truly,

$r \underset{,}{=}$ those wherein they disagree,

$w \underset{,}{=}$ those which both answer truly,

$w' \underset{,}{=}$ those which both answer falsely.

$$w \underset{,}{=} p \,, q \qquad w' \underset{,}{=} \bar{p} \,, \bar{q} \qquad r \underset{,}{=} p \underset{,}{+} q - w \underset{,}{=} \bar{p} \underset{,}{+} \tilde{q} - w'.$$

Now by (28.)

$$p \underset{,}{+} q = p + q - w \qquad \bar{p} \underset{,}{+} \tilde{q} = p - p + 1 - q - w'.$$

Substituting and transposing,

$$2w = p + q - r \qquad 2w' = 2 - p - q - r.$$

$$\text{Now } w_{1-r} = \frac{w \,, (1-r)}{1-r} \qquad \text{but } w \,, (1-r) \underset{,}{=} w.$$

$$w'_{1-r} = \frac{w', (1-r)}{1-r} \qquad \text{but } w', (1-r) = w'$$

$$\therefore w_{(1-r)} = \frac{p + q - r}{2\,(1-r)} \qquad w'_{(1-r)} = \frac{2 - p - q - r}{2\,(1-r)}.$$

The differences of Boole's system, as given by himself, from the modification of it given here, are three.

1st. Boole does not make use of the operations here termed logical addition and subtraction. The advantages obtained by the introduction of them are three, viz. they give unity to the system; they greatly abbreviate the labor of working with it; and they enable us to express *particular* propositions. This last point requires illustration. Let i be a class only determined to be such that only some one individual of the class a comes under it. Then $a \underset{,}{-} i \,, a$ is the expression for some a. Boole cannot properly express some a.

2d. Boole uses the ordinary sign of multiplication for logical multiplication. This debars him from converting every logical identity into an equality of probabilities. Before the transformation can be made the equation has to be brought into a particular form, and much labor is wasted in bringing it to that form.

3d. Boole has no such function as a_b. This involves him in two difficulties. When the probability of such a function is required, he can only obtain it by a departure from the strictness of his system. And on account of the absence of that symbol, he is led to declare that, without adopting the principle that simple, unconditioned events whose probabilities are given are independent, a calculus of logic applicable to probabilities would be impossible.

The question as to the adoption of this principle is certainly not one of words merely. The manner in which it is answered, however, partly determines the sense in which the term "probability" is taken.

In the propriety of language, the probability of a fact either is, or solely depends upon, the strength of the argument in its favor, supposing all relevant relations of all known facts to constitute that argument. Now, the strength of an argument is only the frequency with which *such* an argument will yield a true conclusion when its premises are true. Hence probability depends solely upon the relative frequency of a specific event (namely, that a certain kind of argument yields a true conclusion from true premises) to a generic event (namely, that that kind of argument occurs with true premises). Thus, when an ordinary man says that it is highly probable that it will rain, he has reference to certain indications of rain, — that is, to a certain kind of argument that it will rain, — and means to say that there is an argument that it will rain, which is of a kind of which but a small proportion fail. "Probability," in the untechnical sense, is therefore a vague word, inasmuch as it does not indicate what one, of the numerous subordinated and co-ordinated genera to which every argument belongs, is the one the relative frequency of the truth of which is expressed. It is usually the case, that there is a tacit understanding upon this point, based perhaps on the notion of an *infima species* of argument. But an *infima species* is a mere fiction in logic. And very often the reference is to a very wide genus.

The sense in which the term should be made a technical one is that which will best subserve the purposes of the calculus in question. Now, the only possible use of a calculation of a probability is security in the long run. But there can be no question that an insurance company, for example, which assumed that events were independent without any reason to think that they really were so, would be subjected to great hazard. Suppose, says Mr. Venn, that an insurance company knew that nine tenths of the Englishmen who go to Madeira die, and that nine tenths of the consumptives who go there get well.

How should they treat a consumptive Englishman? Mr. Venn has made an error in answering the question, but the illustration puts in a clear light the advantage of ceasing to speak of probability, and of speaking only of the relative frequency of this event to that.*

Five hundred and eighty-first Meeting.

April 9, 1867. — MONTHLY MEETING.

The PRESIDENT in the chair.

The following paper was presented.

On the Natural Classification of Arguments. By C. S. PEIRCE.

PART I. § 1. *Essential Parts of an Argument.*

In this paper, the term "argument" will denote a body of premises considered as such. The term "premise" will refer exclusively to something laid down, (whether in any enduring and communicable form of expression, or only in some imagined sign,) and not to anything only *virtually* contained in what is said or thought, and also exclusively to that part of what is laid down which is (or is supposed to be) relevant to the conclusion.

Every inference involves the judgment that, if *such* propositions as the premises are are true, then a proposition related to them, as the conclusion is, must be, or is likely to be, true. The principle implied in this judgment, respecting a genus of argument, is termed the *leading principle* of the argument.

A *valid* argument is one whose leading principle is true.

In order that an argument should determine the necessary or probable truth of its conclusion, both the premises and leading principle must be true.

§ 2. *Relations between the Premises and Leading Principle.*

The leading principle contains, by definition, whatever is considered requisite besides the premises to determine the necessary or probable truth of the conclusion. And as it does not contain in itself the subsumption of anything under it, each premise must, in fact, be equivalent to a subsumption under the leading principle.

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* See a notice, Venn's *Logic of Chance,* in the North American Review for July, 1867.